1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part 1, Class XI)
Module Name/Title	Unit 1, Module 3, **Errors in Measurements** Chapter 2, Units and Measurements
Module Id	Keph_10202_eContent
Pre-requisites	Measurement, system of units.
Objectives	**After going through this module, the learners will be able to:** • Understand the meaning of accuracy and precision in measurement • Distinguish between different types of errors and recognize the sources of errors in the measurement, • Learn the various methods of analysis of the errors. • Appreciate the meaning of significant figures in measurement • Focus on significant figures while calculating.
Keywords	Accuracy, Precision, Errors, Systematic errors, Random errors, Significant figures.

2. Development Team

Role	Name	Affiliation
National MOOC Coordinator (NMC)	Prof. Amarendra P. Behera	Central Institute of Educational Technology, NCERT, New Delhi
Programme Coordinator	Dr. Mohd. Mamur Ali	Central Institute of Educational Technology, NCERT, New Delhi
Course Coordinator / PI	Anuradha Mathur	Central Institute of Educational Technology, NCERT, New Delhi
Subject Matter Expert (SME)	Vandita Shukla	Kulachi Hansraj Model School, Ashok Vihar, New Delhi
Review Team	Associate Prof. N.K. Sehgal (Retd.) Prof. V. B. Bhatia (Retd.) Prof. B. K. Sharma (Retd.)	Delhi University Delhi University DESM, NCERT, New Delhi

TABLE OF CONTENTS

1. UNIT SYLLABUS

PHYSICAL WORLD AND MEASUREMENT

Chapter-1: Physical world

Physics- scope and excitement; Nature of physical laws; Physics, technology and society.

Chapter-2: Units and Measurements

Need for measurement: Units of measurement; Systems of units; SI units, fundamental and derived units; Length, Mass and Time measurements; Accuracy and Precision of measuring instruments; errors in measurements; significant figures.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 4 Modules

This unit is divided into four modules for better understanding .

Module 1	• Physical world • Meaning of physics. • Scope and Excitement of physics.
Module 2	• Need of measurement • SI units • fundamental and derived units • Measurement of mass ,length and time
Module 3	• Accuracy, precision • Significant figures • Errors
Module 4	• Expressing physical quantities dimensionally • Dimensional analysis • Application of dimensional analysis

MODULE 3

3. WORDS YOU MUST KNOW

Let us remember the words we have been using in our study of this physics course.

- Physical quantity: All those quantities which can be measured directly or indirectly are called physical quantities.

- Measurement: The measurement of a physical quantity is the process of comparing this quantity with a standard amount of the physical quantity of the same kind, called its unit.

- Base unit: the limited number of units for expressing all the physical quantities. Seven unit of length, mass, time, electric current, temperature, amount of substance and luminous intensity are called vase units

- Derived unit the units of all the physical quantities can be expressed as combinations of base units. Such units obtained for the derived quantities are called derived units. Area, speed are examples of physical quantities whose units would be derived from the base units

- The international system of units: the system of units which is at present internationally accepted for measurement, it is called Systeme Internationale d' unites (French for international system of units). This is abbreviated as SI units.

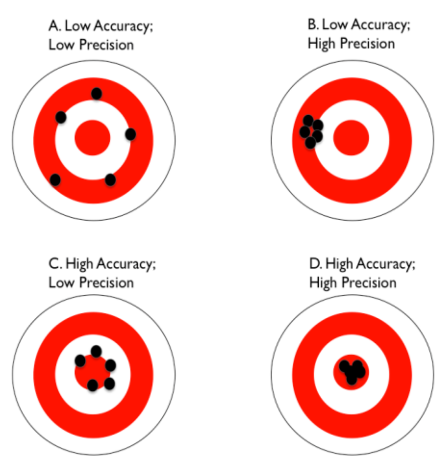
4. INTRODUCTION

Measurement is the foundation of all experimental science and technology. We use different kinds of instruments for measuring various quantities. The result of every measurement by any measuring instrument contains some uncertainty. This uncertainty is called **error**. Every calculated quantity which is based on measured values also has an error. An error can arise due to multiple factors.

This module covers the types of errors, combination of errors and the concept of significant figures.

ACCURACY AND PRECISION

In everyday language "accurate" and "precise"" mean roughly the same thing... but *not* in physics.



To understand the difference between the two terms, let us use the analogy of a shooter who uses a gun to fire bullets at a target. In this analogy, the gun is the instrument, the shooter is the user of the instrument, and the results are determined by the location of the bullet holes in the target.

Bullets are fired at a target, and measurements are taken in relation to the bull's eye at the center of the target. Accuracy describes how close the bullets are to the bull's-eye. The closer a bullet is to the bull's eye, the more accurate is the shot.

How precise the shots are depends on how often the bullets land near each other on the target. When all or most bullets are grouped tightly together, the shots fired can be considered precise since they all landed near the same spot, if not necessarily near the bull's-eye. This is how results can indicate precision but not necessarily accuracy. However, it is important to note that it is not possible to reliably achieve accuracy without precision.

Accurate hits fall close to the bull's-eye, as shown in boards C and D. Precise hits are characterized by the close clustering of consistent hits, shown in boards B and D.

Let's look at another example.

If a basketball player is taking shots, he is accurate if his aim always takes the ball close to, or into, the basket. If his aim takes the ball to the same location each time, whether or not it's close to the basket, he is precise. **A great basketball player is accurate and precise** by shooting the ball exactly the same way each time and getting it into the basket.

Now, let's discuss the meaning of accuracy and precision of instruments in physics.

5. ACCURACY, PRECISION OF INSTRUMENTS

Here, we shall distinguish between two important terms related to measurement i.e.: **Accuracy** and **Precision**.

MEANING OF ACCURACY

The accuracy of a measurement is the measure of how close the measured value is to the true value of the quantity. The accuracy in measurement may depend on several factors, including the limit or the resolution of the measuring instrument. When we reduce the errors, the measurement becomes more accurate.

FOR EXAMPLE

Let the true value of a physical quantity, say thickness of a physics book, is 2.5 cm. When two students were asked to measure it, they measured it as 2.2 cm and 2.4 cm. So in this case 2.4 cm is more accurate as it is closer to the true value.

MEANING OF PRECISION

Precision tells us to what resolution or limit the quantity is measured.

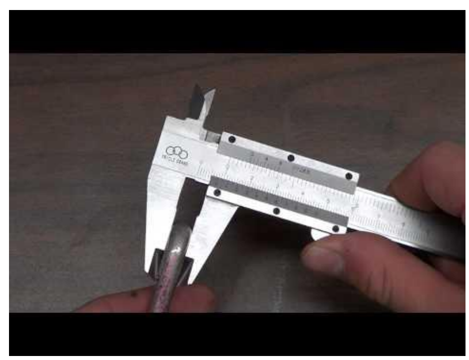
FOR EXAMPLE

The two students again measure the thickness of their physics book by two different devices, meter scale having resolution 0.1 cm and vernier caliper having resolution 0.01cm. They obtain the measured value as 2.4 cm and 2.34 cm. here 2.34 cm is more precise than 2.4 cm. It is not necessary that a more precise value is more accurate as well.

That means in this case we can say that 2.4 cm is less precise but more accurate as it is closer to the true value.

Let's take one more **example:**

Suppose the true value of a certain length is near 3.678 cm. In one experiment, using a measuring instrument of resolution 0.1 cm, the measured value is found to be 3.5 cm, while in another experiment using a measuring device of greater resolution, say 0.01 cm, the length is determined to be 3.38 cm. The first measurement has more accuracy (because it is closer to the true value) but less precision (its resolution is only 0.1 cm), while the second measurement is less accurate but more precise.



Vernier caliper

Thus, every measurement is approximate due to errors in measurement.

The following link is about the difference between accuracy and precision.

https://www.boundless.com/chemistry/textbooks/boundless-chemistry- textbook/introduction-to-chemistry-1/me

The following video is about the difference between accuracy and precision.
https://www.youtube.com/watch?v=mn1BcZKirXQ

6. ERRORS IN MEASUREMENT

While taking observations with some instrument, some uncertainty gets introduced in the measurement. As a result, the measured value is different from the actual or true value .The errors in a measurement are equal to the difference between the true value and the measured value of the quantity.

In general, the errors in measurement can be broadly classified as

(a) **Systematic errors and**

(b) Random errors.

SYSTEMATIC ERRORS

The **systematic errors** are those errors that tend to be in one direction, either positive or negative. Some of the sources of systematic errors are:

(a) **Instrumental errors**

Errors that arise from the errors due to imperfect design or calibration of the measuring instrument zero error in the instrument, etc. For example, the temperature graduations of a thermometer may be inadequately calibrated (it may read 104 °C at the boiling point of water at STP whereas it should read 100 °C); in a Vernier calipers the zero mark of Vernier scale may not coincide with the zero mark of the main scale, or simply an ordinary metre scale may be worn off at one end.

(b) **Imperfection in experimental technique or procedure**

To determine the temperature of a human body, a thermometer placed under the armpit will always give a temperature lower than the actual value of the body temperature. Other external conditions (such as changes in temperature, humidity, wind velocity, etc.) during the experiment may systematically affect the measurement.

(c) Personal errors

Errors that arise due to an individual's bias, lack of proper setting of the apparatus or individual's carelessness in taking observations without observing proper precautions, etc. For example, if you, by habit, always hold your head a bit too far to the right while reading the position of a needle on the scale, you will introduce an error due to **parallax**.

Systematic errors can be minimized by improving experimental techniques, selecting better instruments and removing personal bias as far as possible. For a given set-up, these errors may be estimated to a certain extent and the necessary corrections may be applied to the readings.

(d) Random errors

The **random errors** are those errors, which occur irregularly and hence are random with respect to sign and size. These can arise due to random and unpredictable fluctuations in experimental conditions (e.g. unpredictable fluctuations in temperature, voltage supply, mechanical vibrations of experimental set-ups, etc), personal (unbiased) errors by the observer taking readings, etc. For example, when the same person repeats the same observation, it is very likely that he may get different readings every time.

(e) Least count error

The smallest value that can be measured by the measuring instrument is called its least count.

Every measuring instrument must have a least count. In reliable instruments this value is the same throughout the instrument.

All the readings or measured values are good only up to this value.

The **least count error** is the error associated with the resolution of the instrument. For example, a Vernier caliper has the least count as 0.01cm; a spherometer/ travelling microscope may have a

least count of 0.001 cm. Least count error belongs to the category of random errors but within a limited size; it occurs with both systematic and random errors. If we use a metre scale for measurement of length, it may have graduations at 1 mm division scale spacing or interval.
Using instruments of higher precision, improving experimental techniques, etc., we can reduce the least count error. Repeating the observations several times and taking the arithmetic mean of all the observations, the mean value would be very close to the true value of the measured quantity.

The following link is about systematic and random errors.

https://en.wikipedia.org/wiki/Observational_error

The following video is about systematic and random errors.

https://www.youtube.com/watch?v=awRuUQM_WuM

ABSOLUTE ERROR, RELATIVE ERROR AND PERCENTAGE ERROR

In spite of taking measurement with taking proper precautions into account, all measurements are liable to have errors as discussed above. While dealing with large number of careful observations we calculate the following errors.

Suppose the values obtained in several measurements are a_1, a_2, a_3...., a_n.

The arithmetic mean or average of these values is taken as the best possible value of the quantity under the given conditions of measurement as:

$$\mathbf{a_{mean}} = \frac{(\mathbf{a_1} + \mathbf{a_2} + \mathbf{a_3} + \ldots + \mathbf{a_n})}{\mathbf{n}}$$

a) ABSOLUTE ERROR

The magnitude of the difference between the individual measurement and the true value of the quantity is called the absolute error of the measurement.

This is denoted by $|\Delta \mathbf{a}|$.

In absence of any other method of knowing true value, we considered arithmetic mean as the true value.

Then the errors in the individual measurement values from the true value are

$\Delta a_1 = a_1 - a_{mean}$,

$\Delta a_2 = a_2 - a_{mean}$,

....

....

$\Delta a_n = a_n - a_{mean}$

The Δa calculated above may be positive in certain cases and negative in some other cases. But absolute error $|\Delta a|$ will always be positive; this is because it indicates the deviation from the expected accurate value, so. Both more than and less than values add up towards error

b) MEAN ABSOLUTE ERROR

The arithmetic mean of all the absolute errors is taken as the final or mean absolute error of the value of the physical quantity a. It is represented by $\mathbf{\Delta a_{mean}}$.

Thus,

$$\Delta \mathbf{a}_{\mathbf{mean}} = \frac{(|\Delta \mathbf{a}_1| + |\Delta \mathbf{a}_2| + |\Delta \mathbf{a}_3| + \ldots + |\Delta \mathbf{a}_n|)}{\mathbf{n}}$$

If we do a single measurement, the value we get may be in the range ($a_{mean} \pm \Delta a_{mean}$)

i.e. $a = a_{mean} \pm \Delta a_{mean}$

or,

$a_{mean} - \Delta a_{mean} \leq a \leq a_{mean} + \Delta a_{mean}$

This implies that any measurement of the physical quantity 'a' is likely to lie between ($a_{mean} + \Delta a_{mean}$) and ($a_{mean} - \Delta a_{mean}$).

(c) RELATIVE ERROR

The relative error is the ratio of the mean absolute error Δa_{mean} to the mean value a_{mean} of the quantity measured.

$$\textbf{relative error} = \frac{\Delta \mathbf{a}_{\mathbf{mean}}}{\mathbf{a}_{\mathbf{mean}}}$$

(d) **PERCENTAGE ERROR**

When the relative error is expressed in per cent, it is called the **percentage error** (δa).
Thus, Percentage error

$$\textbf{percentage error } (\boldsymbol{\delta}\mathbf{a}) = \textbf{relative error} \times \mathbf{100} = \frac{\Delta \mathbf{a}_{\mathbf{mean}}}{\mathbf{a}_{\mathbf{mean}}} \times \mathbf{100}$$

Let us now consider an example.

EXAMPLE

We measure the period of oscillation of a simple pendulum. In successive measurements, The readings turn out to be 2.63 s, 2.56 s, 2.42 s, 2.71s and 2.80 s.
Calculate the absolute errors, relative error or percentage error.

SOLUTION:

The mean period of oscillation of the pendulum

$$= \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

= 2.624 s

= 2.62 s

As the periods are measured to a resolution of 0.01 s, all times are to the second decimal; it is proper to put this mean period also to the second decimal.

The **absolute errors** in the measurements are

2.63 s – 2.62 s = 0.01 s

2.56 s – 2.62 s = – 0.06 s

2.42 s – 2.62 s = – 0.20 s

2.71 s – 2.62 s = 0.09 s

2.80 s – 2.62 s = 0.18 s

Note that the errors have the same units as the quantity to be measured.

Mean absolute error

The arithmetic mean of all the absolute errors (for arithmetic mean, we take only the magnitudes) is

ΔT_{mean} = [(0.01+ 0.06+0.20+0.09+0.18) s]/5

= 0.54 s/5

= 0.11 s

That means, the period of oscillation of the simple pendulum is (2.62 ± 0.11) s i.e. it lies between (2.62 + 0.11) s and (2.62 – 0.11) s or between 2.73 s and 2.51 s. As the arithmetic mean of all the absolute errors is 0.11 s, there is already an error in the tenth of a second. Hence there is no point in giving the period to a hundredth. A more correct way will be to write

T = 2.6 ± 0.1 s

The relative error is

δT = 0.1/2.6

= 0.04

The **percentage error** is

= 0.1/2.6 × 100 %

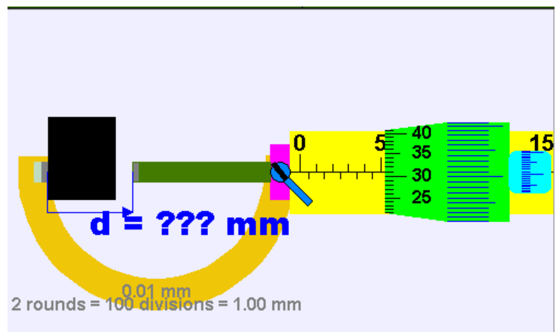
= 4%

EXAMPLE

Watch the video to understand how to take reading using screw gauge.

https://www.youtube.com/watch?v=YAmn-xksu2s

The diameter of a wire as measured by a screw gauge was found to be: 0.026 cm, 0.028 cm, 0.029 cm, 0.027 cm, 0.024 cm and 0.027 cm.



Calculate:

(a) **Mean value of the diameter**

(b) **Mean absolute error**

(c) **Relative error**

(d) **Percentage error**

Also, express the result in terms of absolute error and percentage error.

SOLUTION:

The mean diameter of the wire = $\frac{0.026+0.028+0.029,+0.027+0.024+0.027}{6}$

= 0.027 cm

The m**agnitude of absolute error** in each measurements is:

0.026–0.027 = 0.01cm

0.028–0.027 = 0.01 cm

0.029–0.027 = 0.02cm

0.027–0.027 = 0.00 cm

0.024–0.027 = 0.03 cm

0.027–0.027 = 0.00 cm

Note that: the errors have the same units as the quantity to be measured.

Mean absolute error

The arithmetic mean of all the absolute errors (for arithmetic mean, we take only the magnitudes) is

$$\Delta D_{mean} = [(0.01+ 0.01+0.02 +0.00+0.13 +0.00)\ cm]/6$$

$$= 0.001\ cm$$

Diameter of the wire in terms of the absolute error

$D = (0.027 \pm 0.001)$ cm

The relative error is

$\delta D = 0.001/0.027$

$= 0.04$

The percentage error is

$0.001/0.027 \times 100$ %

= 4%

Diameter of the wire in terms of the percentage error

D = (0.027cm ± 4%)

COMBINATION OF ERRORS

If we do an experiment involving several measurements, we must know how the errors in all the measurements combine. For example, density is obtained by dividing mass by the volume of the substance. If we have errors in the measurement of mass and of the sizes or dimensions, we must know what the error will be in the density of the substance. To make such estimates, we should learn how errors combine in various mathematical operations. For this, we use the following procedure.

(a)Error of a sum or a difference

Suppose two physical quantities A and B have measured values A ± ΔA, B ± ΔB respectively where ΔA and ΔB are their absolute errors. We wish to find the error ΔZ in the sum

Z = A + B.

We have by addition,

Z ± ΔZ = (A ± ΔA) + (B ± ΔB).

The maximum possible error in Z:

ΔZ = ΔA + ΔB

For the difference Z = A – B, we have

Z ± Δ Z = (A ± ΔA) – (B ± ΔB)

= (A – B) ± ΔA ± ΔB

or, ± ΔZ = ± ΔA ± ΔB

The maximum value of the error:

ΔZ = ΔA + ΔB.

Hence the rule:

When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.

EXAMPLE

Two resistors of resistances R_1 = 100 ± 3 ohm and R_2 = 200 ± 4 ohm are connected in series. Find the equivalent resistance of the series combination.

SOLUTION:

The equivalent resistance of series combination

$R = R_1 + R_2$

$= (100 \pm 3)$ ohm + (200 ± 4) ohm

= 300 ± 7 ohm.

EXAMPLE

The temperatures of two bodies measured by a thermometer are t_1 = 20 0C ± 0.5 0C and t_2 = 50 0C ± 0.5 0C.

Calculate the temperature difference and the error therein.

SOLUTION :

Difference in temperature

$t' = t_2 - t_1$

$= (50^0C \pm 0.5^0C) - (20^0C \pm 0.5\ ^0C)$

$\mathbf{t' = 30\ ^0C \pm 1\ ^0C}$

ERROR OF A PRODUCT OR A QUOTIENT

Suppose Z = AB and the measured values of A and B are A ± ΔA and B ± ΔB. Then

$Z \pm \Delta Z = (A \pm \Delta A)(B \pm \Delta B)$

$= AB \pm B\,\Delta A \pm A\,\Delta B \pm \Delta A\,\Delta B.$

Dividing LHS by Z and RHS by AB we have,

$1 \pm (\Delta Z/Z) = 1 \pm (\Delta A/A) \pm (\Delta B/B) \pm (\Delta A/A)(\Delta B/B).$

Since ΔA and ΔB are small, we shall ignore their product.

Hence the maximum relative error

$\Delta Z/ Z = (\Delta A/A) + (\Delta B/B)$.

You can easily verify that this is true for division also.

Let $Z = A/B$

$Z \pm \Delta Z = (A \pm \Delta A)/ (B \pm \Delta B)$

$1 \pm (\Delta Z/Z) = (1 \pm (\Delta A/A))/ (1\pm \Delta B/B)$

$= (1 \pm (\Delta A/A)) (1\pm \Delta B/B)^{-1}$

$1 \pm (\Delta Z/Z) = 1 \pm (\Delta A/A) \pm (\Delta B/B) \pm (\Delta A/A)(\Delta B/B)$.

Since ΔA and ΔB are small, we shall ignore their product.

Hence the maximum relative error

$\Delta Z/ Z = (\Delta A/A) + (\Delta B/B)$.

Hence the rule:

When two quantities are multiplied or divided, the relative error in the result is the sum of the relative errors in the multipliers.

EXAMPLE

The voltage across an electric bulb is (3 ± 0.1) V and the current flowing through it is (2 ± 0.2) A. Find the percentage error in the power consumed.

SOLUTION:

Power = voltage × current

$P = V \times I$

= 3 × 2

= 6 watt

$\Delta P/P \times 100 = \Delta V/V \times 100 + \Delta I/I \times 100$

$\Delta P/P \times 100 = (0.1/3 \times 100 + 0.2/2 \times 100)$

$= (0.03 \times 100 + 0.1 \times 100)$

= 13 %

EXAMPLE

The resistance R = V/I where V = (100 ± 5) V and I = (10 ± 0.2) A. Find the percentage error in R.

SOLUTION:

As $R = \frac{V}{I}$

$$\left(\frac{\Delta R}{R} \times 100\right)_{max} = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$$

$$= \frac{5}{100} \times 100 + \frac{0.2}{10} \times 100$$

= 7%

Error in case of a measured quantity has to be raised to a power for calculation , like in calculation of area or volume

Suppose

$Z = A^2$,

Then,

$\Delta Z/Z = (\Delta A/A) + (\Delta A/A) = 2\,(\Delta A/A)$.

Hence, the relative error in A^2 is two times the error in A.

In general, if $Z = A^p B^q/C^r$

Then,

$\Delta Z/Z = p (\Delta A/A) + q (\Delta B/B) + r (\Delta C/C)$.

Hence the rule:

The relative error in a physical quantity raised to the power k is the k times the relative error in the individual quantity.

EXAMPLE

Find the relative error in Z, if $Z = A^4 B^{1/3} C D^{3/2}$.

SOLUTION:

The relative error in Z is $\Delta Z/Z = 4(\Delta A/A) + (1/3) (\Delta B/B) + (\Delta C/C) + (3/2) (\Delta D/D)$.

EXAMPLE

The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1 s resolution. What is the accuracy in the determination of g?

SOLUTION:

$$g = \frac{4\pi^2 L}{T^2}$$

Here, $T = t/n$ and $(\Delta T) = \Delta t/n$. Therefore, $(\Delta T/T) = \Delta t/t$.

The errors in both L and t are the least count errors. Therefore,

$(\Delta g/g) = (\Delta L/L) + 2(\Delta T/T)$

= 0.1/20.0 + 2 (1/90)

= 0.032

Thus, the percentage error in g is

100 (Δg/g) = 100(ΔL/L) + 2 × 100 (ΔT/T)

= 3%

EXAMPLE

In an experiment, the following observations were recorded:

L = 2.890 cm

M =3.00 kg

l = 0.087 cm

Diameter, D = 0.082 cm

Taking g = 9.81 m/s^2.

Using the formula: Y = $\frac{4\,M\,g\,L}{\pi D^2\,l}$

Find the percentage error in Y.

SOLUTION:

$$Y = \frac{4\,M\,g\,L}{\pi D^2\,l}$$

$$\frac{\Delta Y}{Y} \times 100 = \left(\frac{\Delta M}{M} + \frac{\Delta g}{g} + \frac{\Delta L}{L} + \frac{2\,\Delta D}{D} + \frac{\Delta l}{l}\right) \times 100$$

= (0.01/3.00+0.001/2.890 + 2 x 0.001/0.082 + 0.001/0.087) x 100

= 3.95 %

The following video is about combination of errors.

https://www.youtube.com/watch?v=nrYt08Vi9u4

7. SIGNIFICANT FIGURES

As every measurement involves errors. Thus, the result of measurement should be reported in a way that indicates the precision of measurement. Normally, the reported result of measurement is a number that includes all digits in the number that are known reliably plus the first digit that is uncertain. The reliable digits plus the first uncertain digit are known as **significant digits** or **significant figures**.

For example, if we say the period of oscillation of a simple pendulum is 1.62 s, the digits 1 and 6 are reliable and certain, while the digit 2 is uncertain. Thus, the measured value has three significant figures.

Here are the rules for determining the number of significant figures in a measurement.

RULES TO FIND SIGNIFICANT FIGURES:

1. **All the non-zero digits are significant.**

Example: 1.328 has four significant figures.

6.8956 has five significant figures.

2. **All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.**

Example: 200.7 have four significant figures.

7.6204 have five significant figures.

3. If the number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant.

Example: 0.0445 has three significant figures.

0.648 has three significant figures.

4. The terminal or trailing zero(s) in a number without a decimal point are not significant.

Example: 320400 has four significant figures

640 have two significant figures.

[Thus 123 m = 12300 cm = 123000 mm has three significant figures, the trailing zero(s) being not significant.]

5. The trailing zero(s) in a number with a decimal point are significant.

EXAMPLE:

6.23 0 has four significant figures.

433.00 have five significant figures.

742.000 have six significant figures.

There can be some confusion regarding the trailing zero(s).

Suppose a length is reported to be 4.700 m. It is evident that the zeroes here are meant to convey the precision of measurement and are, therefore, significant. [If these were not, it would be superfluous to write them explicitly, the reported measurement would have been simply 4.7 m].

Now suppose we change units, then

4.700 m = 470.0 cm = 4700 mm = 0.004700 km

Since the last number has trailing zero(s) in a number with no decimal, we would conclude that the number has two significant figures, while in fact, it has four significant figures and a mere change of units cannot change the number of significant figures.

To remove such ambiguities in determining the number of significant figures, the best way is to report every measurement in scientific notation (in the power of 10).

In this notation, every number is expressed as a × 10^b, where 'a' is a number between 1 and 10, and 'b' is any positive or negative exponent (or power) of 10. It is often customary to write the decimal after the first digit. Now the confusion mentioned in (a) above disappears:

$4.700 \text{ m} = 4.700 \times 10^2 \text{ cm}$

$= 4.700 \times 10^3 \text{ mm} = 4.700 \times 10^{-3} \text{ km}$

The power of 10 is irrelevant to the determination of significant figures. However, all zeroes appearing in the base number in the scientific notation are significant. Each number in this case has four significant figures.

Thus, in the scientific notation, no confusion arises about the trailing zero(s) in the base number a. They are always significant.

RULES FOR ARITHMETIC OPERATIONS WITH SIGNIFICANT FIGURES

(1) **In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.**

The rule is illustrated by the following examples:

EXAMPLE

Each side of a cube is measured to be 7.203 m. What are the total surface area and the volume of the cube to appropriate significant figures?

SOLUTION:

The number of significant figures in the measured length is 4.

The calculated area and the volume should therefore be rounded off to 4 significant figures.

Surface area of the cube = $6\times(7.203)^2$ m^2

= 311.299254 m^2

= 311.3 m^2

Volume of the cube = $(7.203)^3$ m^3

= 373.714754 m^3

= 373.7 m^3

EXAMPLE:

5.74 g of a substance occupies 1.2 cm^3. Express its density by keeping the significant figures in view.

SOLUTION:

There are 3 significant figures in the measured mass whereas there are only 2 significant figures in the measured volume. Hence the density should be expressed to only 2 significant figures.

Density = mass /volume

= 5.74/1.2 g cm^{-3}

= 4.8 g cm^{-3}

(2) In addition or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places.

For example, the sum of the numbers 436.32 g, 227.2 g and 0.301 g by mere arithmetic addition, is 663.821 g. But the least precise measurement (227.2 g) is correct to only one decimal place. The final result should, therefore, be rounded off to 663.8 g.

Similarly, the difference in length can be expressed as:

0.307 m – 0.304 m

$= 0.003 \text{ m} = 3 \times 10^{-3} \text{ m}$

Note

that: we should not use the rule (1) applicable for multiplication and division and write 664 g as the result in the example of **addition** and 3.00×10^{-3} m in the example of **subtraction**. They do not convey the precision of measurement properly. For addition and subtraction, the rule is in terms of decimal places.

Rounding off the Uncertain Digits

While rounding off measurements, we use the following rules by convention:

(1) If the digit to be dropped is less than 5, then the preceding digit is left unchanged.

EXAMPLE: 3.43 is rounded off to 3.4.

(2) **If the digit to be dropped is more than 5, then the preceding digit is raised by one.**

EXAMPLE: 8.87 is rounded off to 8.9,

(3) **If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.**

EXAMPLE: 14.351 is rounded off to 14.4

(4) If digit to be dropped is 5 or 5 followed by zeros, then preceding digit is left unchanged, if it is even.

EXAMPLE: 4.250 becomes 4.2 on rounding off,

(5) If digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd.

EXAMPLE: 3.350 is rounded off to 3.46

This video is about significant figures:

https://www.youtube.com/watch?v=EMmnVAexvsA

The following link is about significant figures:

https://www.khanacademy.org/math/pre-algebra/decimals-pre-alg/sig-figs-pre-alg/v/significant-figures

8. SUMMARY

In this module we have learnt

- **Accuracy**: The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity.
- **Precision**: Precision tells us to what resolution or limit the quantity is measured.

- **Errors:** The error is the difference between the measured value and the observed value. The errors are classified as systematic and random errors.
- **Systematic error:** The **systematic errors** are those errors that tend to be in one direction, either positive or negative. Some of the sources of systematic errors are: **Instrumental errors** Imperfection **in experimental technique or procedure and personal errors.**

- **Random errors**: The **random errors** are those errors, which occur irregularly and hence are random with respect to sign and size.
- **Error combination in sum or a difference**: When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.
- **Error combination in product or quotient**: When two quantities are multiplied or divided, the relative error in the result is the sum of the relative errors in the multipliers.
- **Significant figures:** The reliable digits plus the first uncertain digit are known as **significant digits** or **significant figures**.
- **Rules for counting the number of significant figures in a measured quantity:**
  - All the non-zero digits are significant.
  - All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.
  - If the number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant.
  - The terminal or trailing zero(s) in a number without a decimal point are not significant.
  - The trailing zero(s) in a number with a decimal point are significant.

- **Significant figures in algebraic operations:**

In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures

In addition or subtraction, the final result should retain as many decimal places as are there in the number with the least decimal places.

- **Rules for rounding off a measurement:**

Preceding digit is raised by 1 if the insignificant digit to be dropped (the underlined digit in this case) is more than 5, and is left unchanged if the latter is less than 5. If the preceding digit is even, the insignificant digit is simply dropped and, if it is odd, the preceding digit is raised by 1.